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Minimization of entropy generation by asymmetric convective cooling

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Abstract

The uniform internal heating of a solid slab and the viscous flow between two parallel walls, are used to illustrate the possibility of minimizing the global entropy generation rate by cooling the external surfaces convectively in an asymmetric way. The known analytic expressions for the temperature field, in the first case, and the velocity and temperature fields, in the second case, are used to calculate the global entropy generation rate explicitly. In dimensionless terms, this function depends on the dimensionless ambient temperature and convective heat transfer coefficients (Biot numbers) of each surface which, in general, are not assumed to be the same. When the Biot numbers for each surface are equal, the entropy generation rate shows a monotonic increase. However, when the Biot numbers are different this function displays a minimum for specific cooling conditions.

1. Introduction

The entropy generation rate has become a useful tool for evaluating the intrinsic irreversibilities associated with a given process or device [1–5]. By determining the conditions under which this function is minimized, it is possible to optimize the operating conditions by reducing the dissipation to a minimum consistent with the physical constraints imposed on the system. Further, it has been recognized that good engineering heat transfer design in problems where either heat transfer augmentation or thermal insulation are required leads to the minimization of entropy generation [1]. While this approach has proven to be very powerful, surprisingly the analysis of some simple problems that may have important consequences has, to our knowledge, not been carried out so far.

In this paper, the entropy generation minimization method is applied to the analysis of two simple problems, namely, the uniform internal heating of a solid slab and a Poiseuille flow between two infinite parallel plane walls of finite thickness. In both cases, the systems exchange heat with the ambient following Newton's cooling law. [1] In order to make the paper self-contained we solve analytically the heat transfer equation for each case (i.e., the solid slab and the flow between parallel walls) with thermal boundary conditions of the third kind. It is assumed that the heat transfer coefficients for each surface are in general different. From the analytic expressions for the temperature field, in the first case, and the velocity and temperature fields, in the second one, the local and global entropy generation rates are calculated. Fixing the dimensionless convective heat transfer coefficient (Biot number) in one of the surfaces, an optimum convective heat transfer coefficient for the second surface that minimizes the global entropy generation rate is found. In this way, the conditions for minimum total energy loss due to irreversibilities in these systems are determined. For the flow between parallel plates, the behavior of the local Nusselt number

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[1] For an interesting account of how this law, which is originally due to Fourier and not to Newton, acquired its name, see p. 24 and 25 in Ref. [6]. In all fairness one should refer to it as the Fourier–Newton's cooling law.

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Nomenclature

a	slab width or separation between the planar walls (m)
Bi	Biot number, ah/k
h_e	external convective heat transfer coefficient ($\mathrm{W\,m^{-2}\,K^{-1}}$)
h_{eff}	effective convective heat transfer coefficient ($\mathrm{W\,m^{-2}\,K^{-1}}$), $1/(\delta_w/k_w + 1/h_e)$
h_i	internal convective heat transfer coefficient ($\mathrm{W\,m^{-2}\,K^{-1}}$), $-(k/(T_w - T_b))(\partial T/\partial y')_{y'=a/2}$
k	thermal conductivity ($\mathrm{W\,m^{-1}\,K^{-1}}$)
Nu	local Nusselt number, $h_i a/2k$
p	pressure (kPa)
$\dot{q}$	uniform volumetric heating rate ($\mathrm{W/m^3}$)
$\dot{S}'$	local entropy generation rate per unit length ($\mathrm{W\,m^{-4}\,K^{-1}}$)
$\dot{S}$	dimensionless local entropy generation rate per unit length, $a^2\dot{S}'/k$
$\langle\dot{S}\rangle$	dimensionless global entropy generation rate per unit length, $\int_{-1/2}^{1/2}\dot{S}\,\mathrm{d}y$
T	temperature (K)
T_a	external ambient temperature (K)
T_b	bulk fluid temperature (K) $\int_{-a/2}^{a/2} u' T_f\,\mathrm{d}y' / \int_{-a/2}^{a/2} u'\,\mathrm{d}y'$
u'	axial fluid velocity (m/s)
u	dimensionless axial fluid velocity, u'/U_o
U_o	characteristic velocity of the fluid (m/s), $(a^2/\eta)\mathrm{d}p/\mathrm{d}x$
x'	axial coordinate (m)
x	dimensionless axial coordinate, x'/a
y'	transversal coordinate (m)
y	dimensionless transversal coordinate, y'/a

Greek symbols

δ_w	wall thickness (m)
η	dynamic viscosity of the fluid ($\mathrm{kg\,m^{-1}\,s^{-1}}$)
Θ_f	dimensionless fluid temperature, $k_f(T_f - T_a)/\eta U_o^2$
Θ_{af}	dimensionless external ambient temperature (fluid flow problem), $k_f(T_a)/\eta U_o^2$
Θ_{as}	dimensionless external ambient temperature (solid slab problem), $k_s(T_a)/\dot{q}a^2$
Θ_b	dimensionless bulk fluid temperature, $\int_{-1/2}^{1/2} u(\Theta_f + \Theta_{af})\,\mathrm{d}y / \int_{-1/2}^{1/2} u\,\mathrm{d}y$
Θ_s	dimensionless solid temperature, $k_s(T_s - T_a)/\dot{q}a^2$

Subscripts

1	upper wall or surface
2	lower wall or surface
f	fluid
s	solid
w	wall
opt	optimum

and its asymptotic value for minimum entropy generation conditions is also explored.

2. Steady internal heating in a solid slab

Let us consider a solid slab of thermal conductivity k_s and thickness a which experiences a uniform volumetric heating rate $\dot{q}$. We assume that the slab is placed horizontally such that the upper surface is located at $y' = a/2$ and the lower surface is at $y' = -a/2$, y' denoting the transversal coordinate. The temperature field reaches a steady state because the surfaces of the slab are bathed by a fluid of fixed temperature T_a. In dimensionless terms, the heat transfer equation for this system reduces to

$$\frac{\mathrm{d}^2\Theta_s}{\mathrm{d}y^2} = -1, \tag{1}$$

where the dimensionless temperature is given by $\Theta_s = k_s(T_s - T_a)/\dot{q}a^2$, with T_s being the temperature of the solid. The dimensionless (spatial) coordinate y is normalized by a. Evidently, the thermal behavior of the system, particularly heat flow irreversibilities, strongly depends on boundary conditions. Here the heat transfer equation is solved using boundary conditions of the third kind that indicate that the normal temperature gradient at any point in the boundary is assumed to be proportional to the difference between the temperature at the surface and the external ambient temperature. Hence, the amount of heat entering or leaving the system depends on the external temperature as well as on the convective heat transfer coefficient. Let us assume that the external fluid streams that wash each surface of the slab are in general different. Then, the convective heat transfer coefficients, although taken to be constant, do not have the same value on both sides. Therefore, Eq. (1) must satisfy the boundary conditions

$$\frac{\mathrm{d}\Theta_s}{\mathrm{d}y} + Bi_{1s}\Theta_s = 0, \quad \text{at } y = \frac{1}{2}, \tag{2}$$

$$\frac{\mathrm{d}\Theta_s}{\mathrm{d}y} - Bi_{2s}\Theta_s = 0, \quad \text{at } y = -\frac{1}{2}, \tag{3}$$

where the Biot numbers $Bi_{1s} = h_{1s}a/k_s$ and $Bi_{2s} = h_{2s}a/k_s$ are the dimensionless expressions of the convective heat transfer coefficients of the upper and lower surfaces, h_{1s} and h_{2s}, respectively.

The solution of Eq. (1) under boundary conditions (2) and (3) is

$$\Theta_s(y, Bi_{1s}, Bi_{2s}) = -\frac{y^2}{2} + C_s y + D_s, \tag{4}$$

where

$$C_s = -\frac{1}{2}\left(\frac{Bi_{1s} - Bi_{2s}}{Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s}}\right),$$

$$D_s = \frac{1}{8}\left[1 + \frac{4}{Bi_{2s}} - \frac{2(2 + Bi_{2s})(Bi_{1s} - Bi_{2s})}{Bi_{2s}(Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s})}\right].$$

Notice that when $h_{1s} = h_{2s}$ the solution (4) reduces to the well-known solution of the steady internal heating problem as given for instance by Bejan (cf. Eq. (2.62) in Ref. [6]).

2.1. Entropy generation rate

In the case of the steady internal heating problem in a solid material, the entropy generation rate must consider irreversibilities caused by heat flow. Its explicit form can be obtained from the energy and entropy balance equations along with Fourier's law for the heat flux. In dimensionless terms, the local entropy generation rate, $\dot{S}$, is given by [7]

$$\dot{S} = \frac{1}{(\Theta_s + \Theta_{as})^2}\left(\frac{d\Theta_s}{dy}\right)^2, \tag{5}$$

where $\dot{S}$ is normalized by k_s/a^2 and the dimensionless ambient temperature is given by $\Theta_{as} = k_s T_a/\dot{q}a^2$. Once $\dot{S}$ is integrated from $y = -1/2$ to $y = 1/2$, the global entropy generation rate per unit length in the axial direction, $\langle\dot{S}\rangle$, is obtained. The explicit result reads

$$\begin{aligned}\langle\dot{S}\rangle = {} & \frac{Bi_{1s}(2 + Bi_{2s})}{2 + Bi_{2s} + 2\Theta_{as}(Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s})} \\ & + \frac{Bi_{2s}(2 + Bi_{1s})}{2 + Bi_{1s} + 2\Theta_{as}(Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s})} \\ & + \frac{E_s}{\sqrt{F_s}}\left[\arctan\left(\frac{Bi_{1s}(2 + Bi_{2s})}{\sqrt{F_s}}\right)\right. \\ & \left. - \arctan\left(\frac{-Bi_{2s}(2 + Bi_{1s})}{\sqrt{F_s}}\right)\right],\end{aligned} \tag{6}$$

where

$$E_s = 4(Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s}),$$

$$\begin{aligned}F_s = {} & -8(Bi_{1s} + Bi_{2s}) - 4[(Bi_{1s} + Bi_{2s})^2 + Bi_{1s}Bi_{2s}(1 + Bi_{1s} \\ & + Bi_{2s})] - Bi_{1s}^2 Bi_{2s}^2 - 8\Theta_{as}(Bi_{1s} + Bi_{2s} + Bi_{1s}Bi_{2s})^2.\end{aligned}$$

Notice that this quantity only depends on the dimensionless parameters Bi_{1s}, Bi_{2s} and Θ_{as}. Since the global entropy generation rate considers the whole dissipation produced by irreversibilities in the system, we can look for values of the parameters that minimize the function $\langle\dot{S}\rangle$. Let us first explore the behavior of $\langle\dot{S}\rangle$ when the Biot numbers of each surface are the same ($Bi = Bi_{1s} = Bi_{2s}$). This corresponds to symmetric convective cooling. Fig. 1 shows the global entropy generation rate as a function of the single Biot number for different values of the dimensionless ambient temperatures, namely, 1, 2 and 3. For instance, $\Theta_{as} = 1$ may correspond to a commercial copper slab with a thickness of 0.05 m at ambient temperature (20 °C) and a heating rate of 4.36×10^7 W/m^3. As can be observed from Fig. 1, for this case the global entropy generation rate is always a monotonous increasing function of Bi and reaches, for a given Θ_{as}, a limiting value as $Bi \to \infty$.

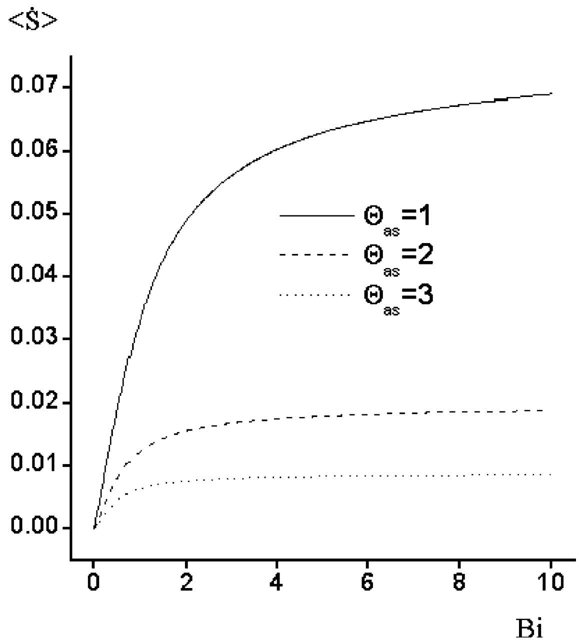

Fig. 1. Global entropy generation rate for the steady internal heating of a solid slab as a function of the single Biot number and different ambient temperatures (symmetric convective cooling).

Let us now consider conditions of asymmetric convective cooling, namely, the case when the Biot numbers for each surface are different. In this case, it is possible to find an optimum Biot number for one of the surfaces which leads to a minimum global entropy generation rate provided the dimensionless ambient temperature and the Biot number of the other surface remain fixed. For instance, if we fix Θ_{as} and the lower surface Biot number, Bi_{2s}, it is found that there is a value of the upper surface Biot number, Bi_{1s}, that minimizes $\langle\dot{S}\rangle$. This is illustrated in Fig. 2 where we have displayed $\langle\dot{S}\rangle$ as a function of Bi_{1s} for $\Theta_{as} = 1$ and different values of Bi_{2s}.

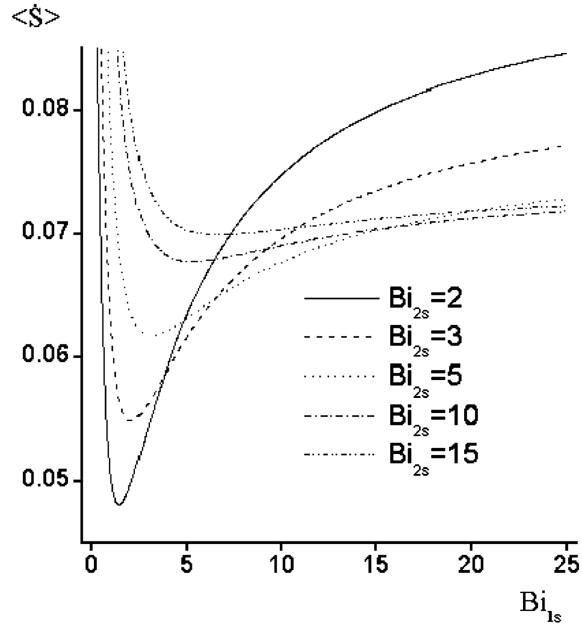

Fig. 2. Global entropy generation rate for the steady internal heating of a solid slab as a function of the upper surface Biot number and different lower surface Biot numbers, $\Theta_{as} = 1$.

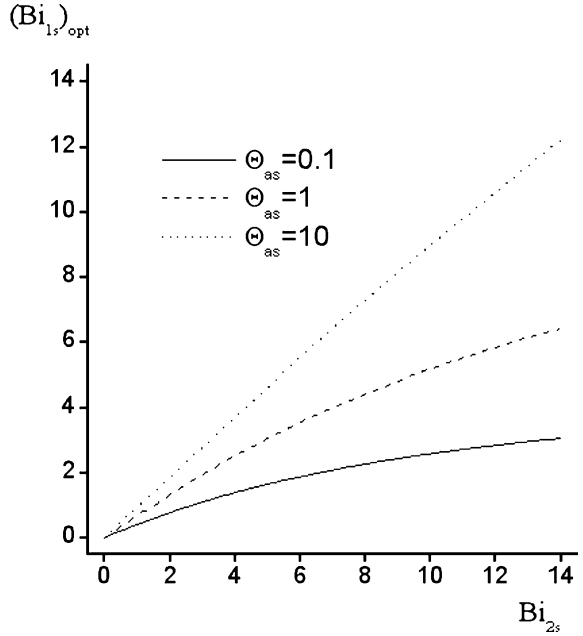

Fig. 3. Optimum upper surface Biot number for the steady internal heating of a solid slab as a function of the lower surface Biot number and different ambient temperatures.

The previous results indicate, therefore, that minimum dissipation can be reached by extracting heat in the system in an asymmetric way. Although we have not been able to derive the value of the minimum analytically, in Fig. 3 we present the numerically determined upper surface optimum Biot number, $(Bi_{1s})_{opt}$, as a function of the lower surface Biot number, Bi_{2s}, for three different values of the dimensionless ambient temperature. The behavior observed in the figure indicates that the lower the ambient temperature the higher the asymmetry in the cooling required to achieve the minimum irreversible losses.

Let us now turn to the analysis of the asymmetric cooling in a problem where apart from heat flow there are irreversibilities due to viscous dissipation.

3. Viscous flow between parallel walls

The dimensionless velocity field in Poiseuille flow between two infinite parallel plane walls located at $y = -1/2$ and $y = 1/2$ is

$$u = \frac{1}{8}(1 - 4y^2), \tag{7}$$

where the velocity, u, has been normalized by $U_o = (a^2/\eta)\mathrm{d}p/\mathrm{d}x$, a denoting in this instance the separation between the walls, and η and $\mathrm{d}p/\mathrm{d}x$ being the dynamic viscosity of the fluid and the imposed pressure gradient, respectively.

With the previous velocity field, we proceed to solve the energy balance equation considering viscous dissipation. Once again, the thermal behavior of the flow strongly depends on boundary conditions. As before, we consider boundary conditions of the third kind. In this case, the amount of heat entering or leaving the system depends on the external temperature as well as on the effective convective heat transfer coefficient which includes both the thermal wall resistance and the external convective heat transfer coefficient. The heat transfer equation, in dimensionless form, reduces to

$$\frac{\mathrm{d}^2\Theta_f}{\mathrm{d}y^2} + \left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2 = 0, \tag{8}$$

where the dimensionless temperature is now given by $\Theta_f = k_f(T_f - T_a)/\eta U_o^2$, with T_f and T_a being the fluid and ambient temperatures, respectively, and k_f the fluid thermal conductivity. Eq. (8) must satisfy the boundary conditions

$$\frac{\mathrm{d}\Theta_f}{\mathrm{d}y} + Bi_1\Theta_f = 0, \quad \text{at } y = \frac{1}{2}, \tag{9}$$

$$\frac{\mathrm{d}\Theta_f}{\mathrm{d}y} - Bi_2\Theta_f = 0, \quad \text{at } y = -\frac{1}{2}, \tag{10}$$

where the Biot numbers $Bi_1 = (h_{eff})_1 a/k_f$ and $Bi_2 = (h_{eff})_2 a/k_f$ are the dimensionless expressions of the convective heat transfer coefficients of the upper and lower walls, $(h_{\mathrm{eff}})_1$ and $(h_{\mathrm{eff}})_2$, respectively, which in general, are assumed to be different. Here

$$(h_{\mathrm{eff}})_j = \frac{1}{\frac{(\delta_{\mathrm{w}})_j}{(k_{\mathrm{w}})_j} + \frac{1}{(h_{\mathrm{e}})_j}}, \quad j = 1, 2. \tag{11}$$

In Eq. (11), δ_{w} and k_{w} are the wall thickness and the wall thermal conductivity, respectively, while $(h_{\mathrm{e}})_1$ and $(h_{\mathrm{e}})_2$ are the external convective heat transfer coefficient of the upper and lower walls, respectively.

The solution for the temperature field is given in the form

$$\Theta_{\mathrm{f}}(y, Bi_1, Bi_2) = -\frac{y^4}{12} + C_{\mathrm{f}} y + D_{\mathrm{f}}, \tag{12}$$

where

$$C_{\mathrm{f}} = -\frac{1}{24}\frac{Bi_1 - Bi_2}{Bi_1 + Bi_2 + Bi_1 Bi_2},$$

$$D_{\mathrm{f}} = \frac{1}{192}\left[1 + \frac{8}{Bi_2} - \frac{4(Bi_1 - Bi_2)(2 + Bi_2)}{Bi_2(Bi_1 + Bi_2 + Bi_1 Bi_2)}\right].$$

We now proceed to calculate the entropy generation rate using the previous velocity and temperature fields.

3.1. Entropy generation rate

In the flow of a monocomponent viscous fluid, the entropy generation rate, $\dot{S}$, can be written explicitly in dimensionless terms as [7]

$$\dot{S} = \frac{1}{(\Theta_{\mathrm{f}} + \Theta_{\mathrm{af}})^2}\left(\frac{\mathrm{d}\Theta_{\mathrm{f}}}{\mathrm{d}y}\right)^2 + \frac{1}{\Theta_{\mathrm{f}} + \Theta_{\mathrm{af}}}\left(\frac{\mathrm{d}u}{\mathrm{d}y}\right)^2, \tag{13}$$

where $\dot{S}$ is normalized by k_{f}/a^2 and the dimensionless ambient temperature is given by $\Theta_{\mathrm{af}} = k_{\mathrm{f}} T_{\mathrm{a}}/\eta U_{\mathrm{o}}^2$. In writing Eq. (13), we have taken into account irreversibilities caused by both viscous dissipation and heat flow. The largest values of $\dot{S}$ are obtained near the walls for it is in these regions where the strongest dissipation occurs. The global entropy generation rate per unit length in the axial direction, $\langle \dot{S} \rangle$, is once more obtained by integrating $\dot{S}$ from $y = -1/2$ to $y = 1/2$. The explicit result reads

$$\langle \dot{S} \rangle = \frac{Bi_1(2 + Bi_2)}{2 + Bi_2 + 24\Theta_{\mathrm{af}}(Bi_1 + Bi_2 + Bi_1 Bi_2)} + \frac{Bi_2(2 + Bi_1)}{2 + Bi_1 + 24\Theta_{\mathrm{af}}(Bi_1 + Bi_2 + Bi_1 Bi_2)}. \tag{14}$$

In this instance, the resulting quantity only depends on the dimensionless parameters Bi_1, Bi_2 and Θ_{af}. One can again look for values of the parameters that minimize the function $\langle \dot{S} \rangle$. As in the previous example, we have first explored the behavior of $\langle \dot{S} \rangle$ when the Biot numbers of each wall are the same ($Bi = Bi_1 = Bi_2$), that is, the symmetric convective cooling of the walls. Although not displayed graphically, similarly to what occurred in the steady internal heating problem, we found that in the present example the global entropy generation rate is always a monotonous increasing function of Bi (with no minima) and eventually reaches a limiting value as $Bi \to \infty$.

When we consider the case where the Biot numbers for each wall are different, i.e. conditions of asymmetric convective cooling, it is possible to find again an optimum Biot number for one of the walls which leads to a minimum global entropy generation rate provided the dimensionless ambient temperature and the Biot number of the other wall remain fixed. In fact, the result may be derived analytically. For instance, if we fix Θ_{af} and the lower wall Biot number, Bi_2, it is found that the value of the upper wall Biot number, Bi_1, that minimizes $\langle \dot{S} \rangle$ is given by

$$\begin{aligned}(Bi_1)_{\mathrm{opt}} = \frac{1}{2\beta}\Big\{ -\alpha + \{\alpha^2 - 16\beta[(Bi_2 + 2)^2 &+ 6\Theta_{\mathrm{af}}(16Bi_2 + 8Bi_2^2 - 2Bi_2^3 - Bi_2^4) \\ &+ 144\Theta_{\mathrm{af}}^2(4Bi_2^2 - Bi_2^4)]\}^{1/2}\Big\},\end{aligned} \tag{15}$$

where

$$\begin{aligned}\alpha = 4[(Bi_2 + 2)^2 &+ 12\Theta_{\mathrm{af}}(8 + 24Bi_2 + 16Bi_2^2 + 3Bi_2^3) \\ &+ 288\Theta_{\mathrm{af}}^2(4Bi_2 + 8Bi_2^2 + 3Bi_2^3)]\end{aligned}$$

and

$$\begin{aligned}\beta = (Bi_2 + 2)^2 &+ 8\Theta_{\mathrm{af}}(24 + 54Bi_2 + 33Bi_2^2 + 6Bi_2^3) \\ &+ 576\Theta_{\mathrm{af}}^2(4 + 16Bi_2 + 19Bi_2^2 + 8Bi_2^3 + Bi_2^4).\end{aligned}$$

Fig. 4 shows the optimum upper wall Biot number, $(Bi_1)_{\mathrm{opt}}$, as a function of Bi_2 for different values of Θ_{af}. It is clear from the figure that as Θ_{af} changes in two orders of magnitude there is only a slight change in the curves. It is found that as Bi_2 increases, $(Bi_1)_{\mathrm{opt}}$ approaches a limiting value which explicitly reads

$$\lim_{Bi_2 \to \infty} (Bi_1)_{\mathrm{opt}} = \left(1 + \frac{1}{24\Theta_{\mathrm{af}}}\right)^{1/2}. \tag{16}$$

On the other hand, it is also observed that for $Bi_2 < 2$, $(Bi_1)_{\mathrm{opt}}$ takes negative values which evidently have no physical meaning. In fact, for $Bi_2 < 2$ no minimum values of $\langle \dot{S} \rangle$ are found. This can be observed in Fig. 5 where the global entropy generation rate as a function of the upper wall Biot number, Bi_1, is shown for different values of the lower wall Biot number ($Bi_2 = 1$, 2, 3, 4 and 6) and $\Theta_{\mathrm{af}} = 7$. This value of Θ_{af} is obtained using the physical properties of engine oil [8] at an ambient temperature $T_{\mathrm{a}} = 20$ °C. For each curve, the function $\langle \dot{S} \rangle$ is normalized by its value at $Bi_1 = 0$. Notice that minimum values of $\langle \dot{S} \rangle$ are observed to occur for a given Bi_1 provided $Bi_2 > 2$. Also, the higher the value of Bi_2, the higher the value of the minimum. Once the minimum

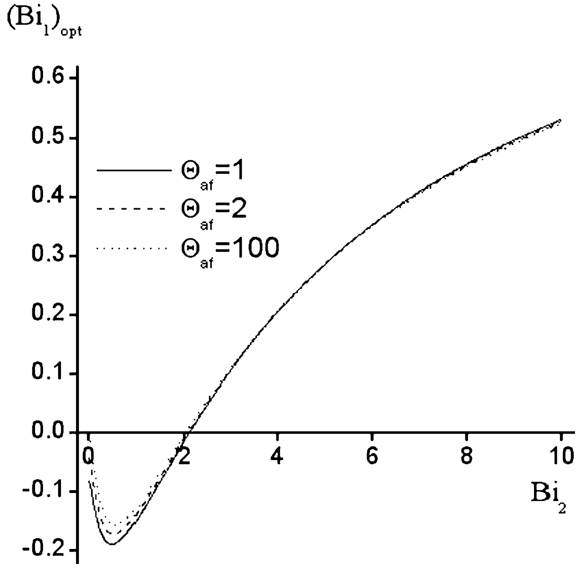

Fig. 4. Optimum upper wall Biot number for viscous flow between parallel walls as a function of the lower wall Biot number and different ambient temperatures.

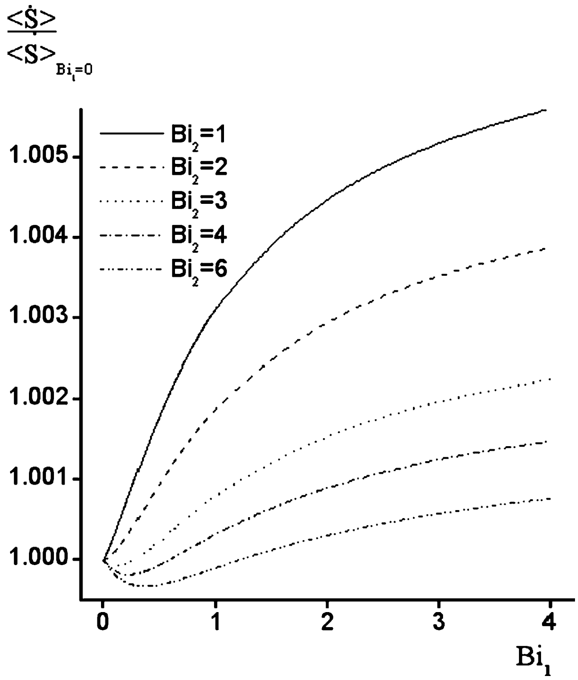

Fig. 5. Normalized global entropy generation rate for viscous flow between parallel walls as a function of the upper wall Biot number and different lower wall Biot numbers, $\Theta_{af} = 7$.

values are reached, $\langle \dot{S} \rangle$ exhibits a monotonic increase as Bi_1 grows. Fig. 6 shows also $\langle \dot{S} \rangle$ versus Bi_1 but for higher values of Bi_2. As Bi_2 increases the optimum value of Bi_1

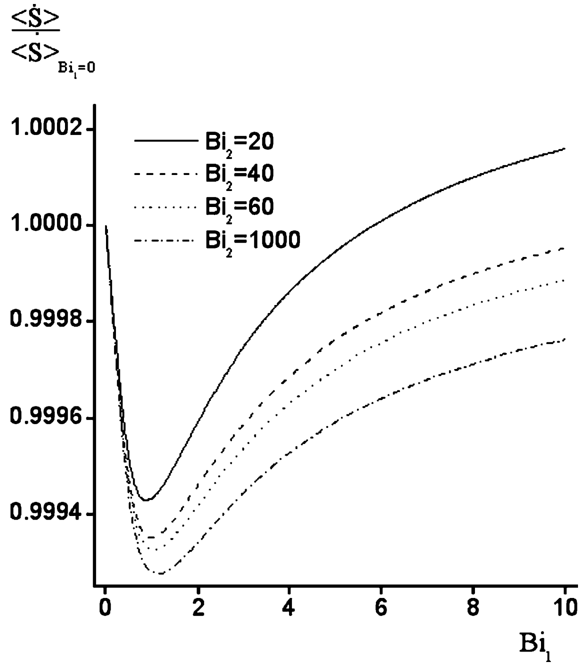

Fig. 6. Normalized global entropy generation rate for viscous flow between parallel walls as a function of the upper wall Biot number and different lower wall Biot numbers, $\Theta_{af} = 7$.

also increases but it reaches the limit given by Eq. (16) as $Bi_2 \to \infty$. The foregoing results indicate, therefore, that once more the minimum dissipation can be reached by extracting heat in the system in an asymmetric way.

Let us now calculate the local Nusselt number at the upper wall, based on the internal convective heat transfer coefficient, h_i, namely [6],

$$h_i = -\frac{k_f}{T_w - T_b}\left(\frac{\partial T_f}{\partial y'}\right)_{y'=a/2}, \tag{17}$$

where T_b and T_w are the dimensional expressions of the bulk temperature (i.e., the cross-section averaged temperature of the stream) and the temperature at the wall, respectively. Hence, the global Nusselt number at the upper wall is given by

$$\begin{aligned} Nu &= \frac{h_i a}{2k_f} = -\frac{(\mathrm{d}\Theta_f/\mathrm{d}y)_{y=1/2}}{2(\Theta_f(y=1/2) + \Theta_{af} - \Theta_b)} \\ &= \frac{Bi_1 \Theta_f(y=1/2)}{2(\Theta_f(y=1/2) + \Theta_{af} - \Theta_b)}, \end{aligned} \tag{18}$$

where the dimensionless bulk temperature is defined as

$$\Theta_b = \frac{\int_{-1/2}^{1/2} u(\Theta_f + \Theta_{af})\,\mathrm{d}y}{\int_{-1/2}^{1/2} u\,\mathrm{d}y}.$$

Fig. 7 shows the Nusselt number (18) evaluated at the optimum upper wall Biot number, $(Bi_1)_{opt}$, as a function

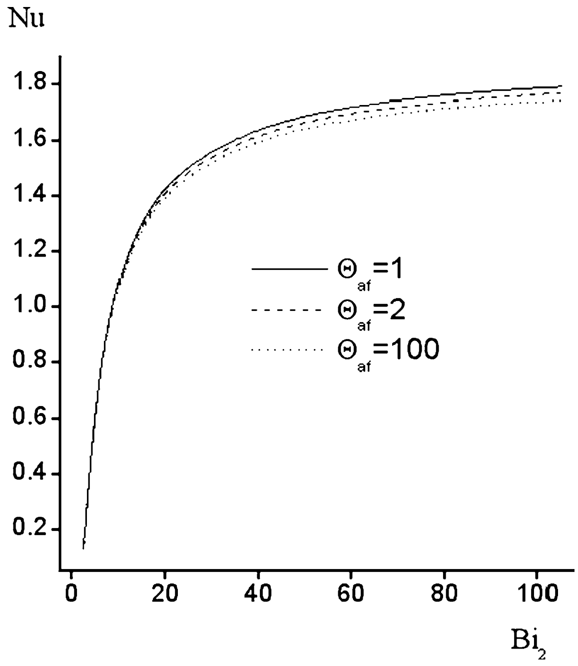

Fig. 7. Local Nusselt number for minimum entropy generation conditions for viscous flow between parallel walls as a function of the lower wall Biot number and different ambient temperatures.

of Bi_2 for different dimensionless ambient temperatures. This local Nusselt number for minimum entropy generation conditions displays a monotonic behavior as Bi_2 increases and reaches a limiting value as $Bi_2 \to \infty$. In fact, this limiting value can be determined analytically, namely,

$$\lim_{Bi_2 \to \infty} (Nu)_{\mathrm{opt}} = \frac{\sqrt{1 + (1/24\Theta_{\mathrm{af}})}}{1 - 0.2286\left(1 + \sqrt{1 + (1/24\Theta_{\mathrm{af}})}\right)} \tag{19}$$

which depends only on the dimensionless ambient temperature. As can be seen from the figure, the effect of the ambient temperature on the upper wall Biot number dependence of the local Nusselt number is not very pronounced.

4. Concluding remarks

In this paper, using two of the simplest heat transfer problems, namely, the steady internal heating of a solid slab and the viscous flow between two parallel plane walls of finite thickness, we have shown that minimum entropy generation rates can be reached by extracting heat in the system in an asymmetric way.

Our point of departure was the solution of the balance equations for these problems yielding in the first case the temperature and in the second, the velocity and temperature fields. In both problems the temperature field was determined applying thermal boundary conditions of the third kind, assuming in general that the convective heat transfer coefficients for the boundaries are different. Under these conditions, and assuming that one of the heat transfer coefficients (say that of the lower surface or wall) and the ambient temperature are fixed, the global entropy generation rate as a function of the other heat transfer coefficient displays a minimum. This provides the conditions under which the irreversibilities due to viscous friction and/or heat flow are minimized. The results for both examples show many similarities but also two important differences. On the one hand, it is somewhat surprising that the apparently simpler situation (i.e., the internal heating of a solid slab) leads to a more complex expression for the global entropy generation rate. This is tied to the fact that the (spatial) integration in the case of Poiseuille flow leads to algebraic expressions whereas that of the solid slab involves trascendental functions. On the other hand, in the internal heating problem there is *always* an optimum upper surface Biot number irrespective of the value of the lower surface Biot number. In contrast, in the viscous flow case, only for $Bi_2 > 2$ the upper wall Biot number producing a minimum in the global entropy generation rate is physically meaningful. Finally, the effect of the ambient temperature on the minimum entropy generation conditions is by far more important in the steady internal heating of a solid slab than in the Poiseuille flow between parallel planar walls.

Although the problems we solved are very simple ones, we are persuaded that the observed trends should also be present in more complex situations. Whether this conjecture is valid requires further work on this subject. For instance, it appears that when nonlinear convective effects in the inner flow are present, a minimum also arises under asymmetric convective cooling. A detailed investigation of this problem is presently under way and will be reported elsewhere.

As remarked earlier, good engineering heat transfer design aims at minimizing the losses due to irreversible behavior. Therefore, the possibility of reaching a minimum in the entropy generation rate using asymmetric convective cooling, which is the central result of this paper, might be useful to optimize operating conditions of heat transfer devices.

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